Direct numerical simulation of three-dimensional coherent structure in plane mixing layer *

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Abstract The three-dimensional temporally evolving plane mixing layer is simulated by directly solving the Navier-Stokes equations using pseudo-spectral method. The process of loss of stability, and the formation paring, and development of vortex are presented. The simulated result shows that the evolving characteristics of coherent structure are important mechanism of growing and entrainment of mixing layer.

Keywords: spectral method, direct numerical simulation, mixing layer, coherent structure.

Plane mixing layer as a classical model for the study of turbulence in free shear layers has been extensively studied in the past decades. But research fields were mostly limited to the basic solutions and stability of laminar flow and turbulence shear flows, etc. With the improvement in experimental techniques and the development of computer science and technology, several investigations have been focused on the detailed structure and evolving characteristics of vortex via experiment, theoretical analysis, and direct numerical simulation. For example, Riley and Metcalfe[1] directly simulated the mixing layer with low Reynolds number and found that the momentum thickness grew linearly approximately with time, and flow fields had similar average velocity profiles. Cain et al.1) introduced coordinate transform to simplify infinite flow region into finite computational region, which showed the process of large scale structures by adopting large eddy simulation. Lasheras and Choil[2] experimentally studied the relations between vortex development and instability, and obtained the results that the characteristic time of instability in two-dimension is far less than that in three-dimension. Besides, Rogers and Moser described numerically the detailed rollup[3] and self-similar layer[4]; Wu and Shi[5] directly simulated the coherent structure of a two-dimensional mixing layer using the method of mapping functions. Fu and Ma[6] introduced high order accuracy discretization and group velocity control to study how coherent structures are affected by the shocks induced by vortex pairing. Although the same conclusions were drawn from numerical simulation, experimental study and theoretical analysis, no thorough dynamical description of mixing layer flow is available. In particular, there is still controversy over the development of three-dimensionality and the transition to turbulence. So this problem

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1) Cain, A. B. et al. A three-dimensional simulations of transition and early turbulence in a time-development mixing layer. Thermosci Div., Rep. TF-14, Dept. Of Mech. Eng., Stanford Univ., 1981, unpublished data.

remains to be further investigated.

The process of vortex rollup and pairing as well as the evolving characteristics of coherent structure is presened through direct numerical simulation of temporally evolving mixing layer. It is of momentous significance to understand the nonlinear development of three-dimensionality.

1 Numerical model

1.1 Governing equations

Figure 1 shows the plane mixing layer consisting of two parallel streams with different velocities U_1 and $U_2(U_1 > U_2)$. The non-dimensional continuity and momentum equations for an incompressible flow with no body force are

$$\frac{\partial \boldsymbol{V}}{\partial t} = \boldsymbol{F} - \nabla \Pi + \frac{1}{Re}\nabla^2 \boldsymbol{V}, \tag{1}$$

$$\nabla \cdot \boldsymbol{V} = 0, \tag{2}$$

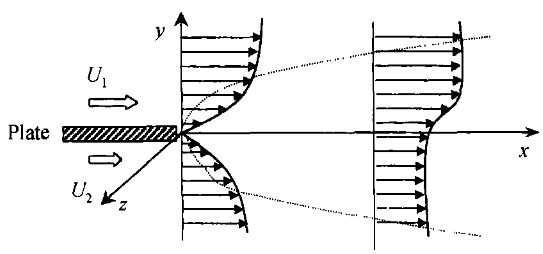

Fig. 1 Schematic of plane mixing layer.

where $\boldsymbol{F} = \boldsymbol{V} \times \boldsymbol{\omega}$, Π is the total pressure, vorticity $\boldsymbol{\omega} = \nabla \times \boldsymbol{V}$, Reynolds number $Re = \bar{U}\theta_0/\upsilon$. $\bar{U} = (U_1 + U_2)/2$ and θ_0 (the initial momentum thickness) will be taken as characteristic velocity and length, respectively.

1.2 Initial conditions

The initial conditions are described as follows. (i) The base flow with hyperbolic-tangent profile is $u_0 = 1 + 0.5\Delta U/\bar{U}\,\mathrm{th}(y)$, where $\Delta U = (U_1 - U_2)$ (ii) The streamfunction of two-dimensional disturbance is

$$\psi(x, y) = A_1 Re(\phi_1(y)\exp\cdot(\mathrm{i}\alpha_1 x)) + A_2 Re(\phi_2(y)\exp(\mathrm{i}\alpha_2 x)),$$

where the amplitudes $A_1 = 0.15$ and $A_2 = 0.08$, Re denotes an operator getting the real part from a complex, $\phi_1(y)$ and $\phi_2(y)$ are the normalized eigenfunctions with respect to α_1 and α_2[7] respectively. Corresponding to the most unstable perturbations, the fundamental wave number α_1 is set to be 0.4446, and thesubharmonic unstable wave number α_2 satisfies the condition of resonance ($\alpha_2 = \alpha_1/2$).

(iii) The streamwise vorticity of initial three-dimensional perturbation is $\omega_x = A_3\exp(-y^2/2)\cdot\sin(\alpha_3 z)$, where amplitude $A_3 = 0.05$, and spanwise wave number $\alpha_3 = 0.67$.

2 Numerical procedure

A pseudo-spectral method is employed to solve the governing equations. In the homogeneous directions (x, z), all the quantities are expressed by Fourier expansions. The periodicity lengths in the streamwise and spanwise directions are $L_x = 2\pi/\alpha_2$ and $L_z = 2\pi/\alpha_3$, respectively. In transverse direction (y), considering that the perturbation attenuates fast in this direction, the solutions in this direction can also be represented by the same expansion after the introduction of mirror image extension. The half periodicity length $L_y = 25$. The integral domain D is defined as $(0, L_x) \times (-L_y/2, L_y/2) \times (0, L_z)$; the number of collocation points in this computational domain is $I \times J \times K = 256 \times 256 \times 64$.

All the quantities in Eq.(2) are expressed by Fourier expansions in x, y and z directions as follows:

$$V(\boldsymbol{x}, t) = \sum_{|k_1| \leqslant I/2} \sum_{|k_2| \leqslant J/2} \sum_{|k_3| \leqslant K/2} \boldsymbol{v}(\boldsymbol{k}, t) \exp(\mathrm{i}\boldsymbol{k\alpha} \cdot \boldsymbol{x}), \tag{3}$$

$$\Pi(\boldsymbol{x}, t) = \sum_{|k_1| \leqslant I/2} \sum_{|k_2| \leqslant J/2} \sum_{|k_3| \leqslant K/2} \pi(\boldsymbol{k}, t) \exp(\mathrm{i}\boldsymbol{k\alpha} \cdot \boldsymbol{x}), \tag{4}$$

$$\boldsymbol{F}(\boldsymbol{x}, t) = \sum_{|k_1| \leqslant I/2} \sum_{|k_2| \leqslant J/2} \sum_{|k_3| \leqslant K/2} \boldsymbol{f}(\boldsymbol{k}, t) \exp(\mathrm{i}\boldsymbol{k\alpha} \cdot \boldsymbol{x}), \tag{5}$$

in which $\boldsymbol{v}(\boldsymbol{k}, t)$, $\pi(\boldsymbol{k}, t)$ and $\boldsymbol{f}(\boldsymbol{k}, t)$ are the Fourier coefficients of velocity $\boldsymbol{V}(\boldsymbol{x}, t)$, total pressure $\Pi(\boldsymbol{x}, t)$ and non-linearity term $\boldsymbol{F}(\boldsymbol{x}, t)$ respectively, and $\boldsymbol{x} = (x, y, z)$, $\boldsymbol{k} = (k_1, k_2, k_3)$, $\boldsymbol{k\alpha} = (2\pi k_1/L_x,\ 2\pi k_2/L_y,\ 2\pi k_3/L_z)$ and $\mathrm{i} = \sqrt{-1}$.

Substitute Eqs.(3) ~ (5) into Eqs.(1) and (2) to obtain all the Fourier coefficients above. Then all spatial quantities can be determined by Fourier reverse transformation. Time advancement of the equations is completed by the two-level explicit Adams-Bashforth scheme for the non-linear term $\boldsymbol{f}(\boldsymbol{k}, t)$ and the implicit Crank-Nicolson method for other terms, and time step is set to be $\Delta t = 0.02$, and $Re = 250$.

3 Results and discussions

Figure 2 shows instantaneous flow field $(\boldsymbol{u}, \boldsymbol{v})$ on the plane $z = L_z/4$. Periodic spanwise vortex rollup resulting from Kelvin-Helmholtz instability can be clearly seen in Fig. 2(a). The periodic structures lose stability again when the perturbations strengthened, then pairing and coalition of vortex occurred as shown in Figure 2 (c,d).

The contours of spanwise vorticity on the plane $z = L_z/4$ is shown in Fig. 3, where the development of the first pairing of two adjacent Kelvin-Helmholtz rollers is caused by the initial perturbations on the subharmonic unstable wave number. During the pairing, a pair of well-developed rollers come together, corotate and eventually amalgamate to form a new bigger roller. The spanwise vortices are depleted in the braid region (the region between pairings) while the pairing is occurring. However, after the pairing, the spanwise vortices are advected back into the braid region, which is called over-

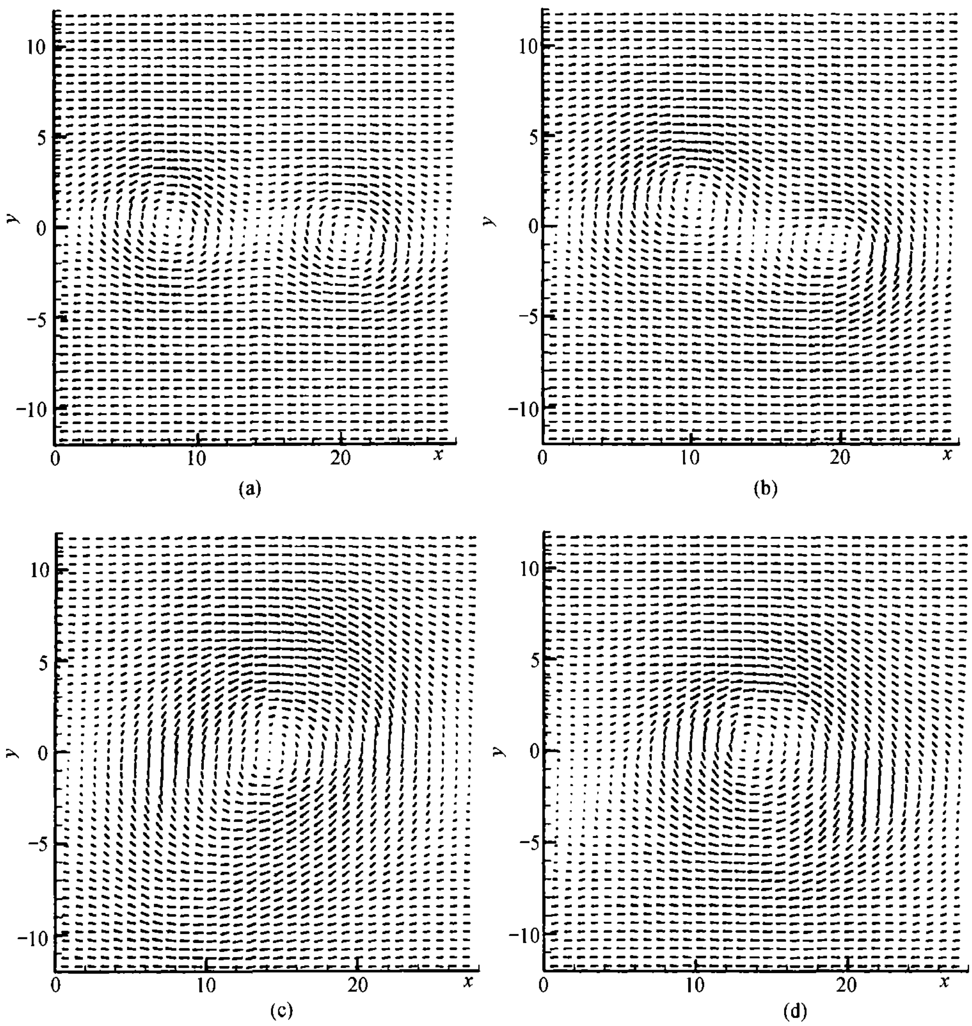

Fig. 2 Instantaneous flow field in the plane $z = L_z/4$. (a) $T = 25$; (b) $T = 45$; (c) $T = 85$; (d) $T = 115$.

saturation.

Figure 4 shows the three-dimensional contours of streamwise vorticity. The solid lines represent the positive values of the vorticity and the dashed lines represent the negativeones. Due to the symmetry of perturbations in spanwise, the figure just shows half periodic length in this direction for an easy view. The mechanism of formation and development of streamwise vorticity is subject to three-dimensional instability. It can be found that counter-rotating streamwise rib vortices first occur in braid region and then extend from the bottom of one roller to the top of the next, which is in good agreement with the experimental observation by Lasheras and Choil[2]. Besides the ribs, the quadrupoles are another kind of notable vortices, which can be clearly observed in the figure when $T = 25$ (dashed line). The quadrupole occurs in the core region with the initial streamwise tube cutting and departing there because of the stretching of the forming rollers. With the time elapsing, two quadrupoles come together, and the rib between them is extruded and turns short ($T = 60$). Afterwards, the streamwise

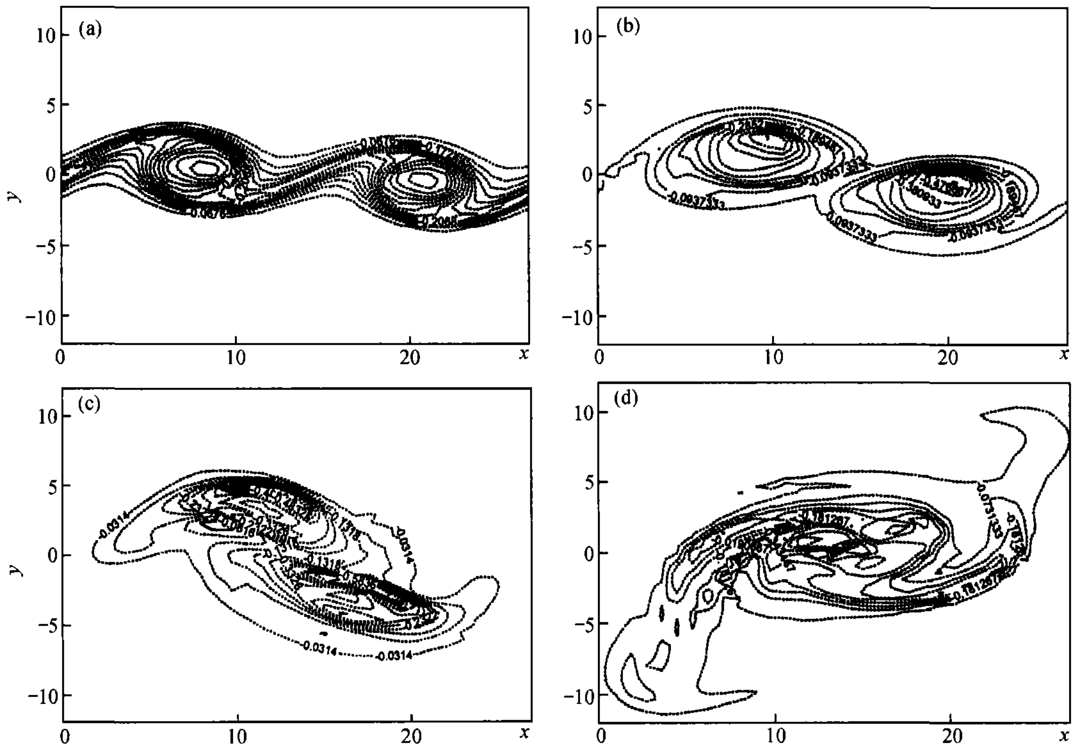

Fig. 3 Contours of spanwise vorticity. (a) $T = 25$, $z = L_z/4$; (b) $T = 45$, $z = L_z/4$; (c) $T = 60$, $z = L_z/4$; (d) $T = 100$, $z = L_z/4$.

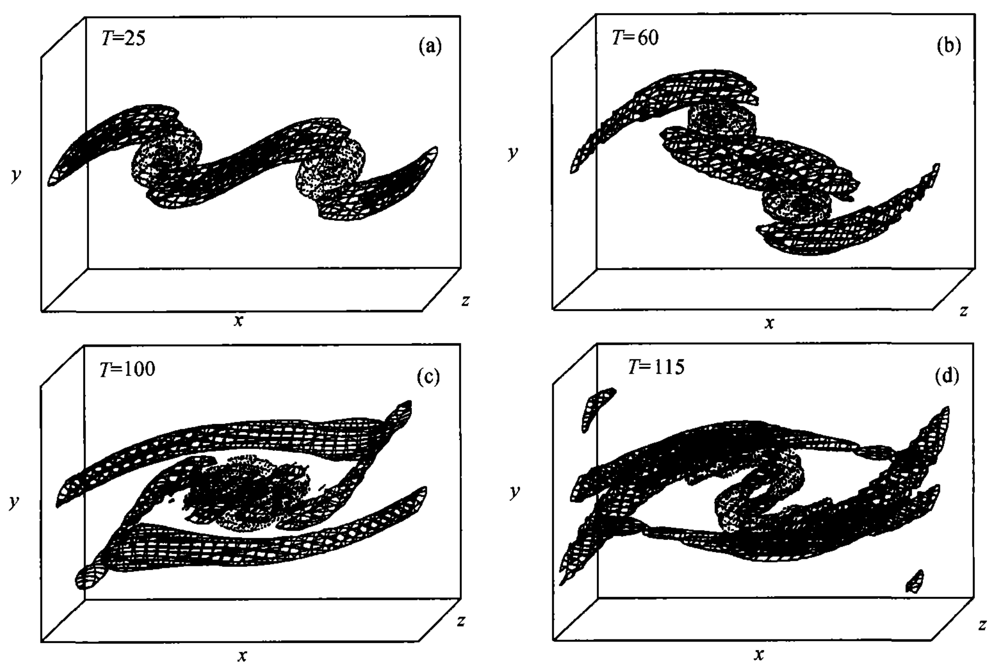

Fig. 4 Three-dimensional contours of streamwise vorticity.

vortex tubes stretch and gradually become approximately vortex tube structures ($T = 100$). Finally, small ribs are induced by larger ones, leading to vortices tubes' gradual amalgamation. In core regions, vortices are also regrouping and forming coherent pairing structures.

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